

WOMBAT 2020

Online Edition

Workshop on Optimisation, Metric Bounds,
Approximation and Transversality

30 November - 4 December 2020



PROGRAM AND ABSTRACTS

The Fifth Workshop on Optimisation, Metric Bounds, Approximation and Transversality (WOMBAT2020) will be held online 30 November – 4 December 2020. This year WOMBAT is dedicated to the memory of Prof Alexander Rubinov.

Website: <https://wombat.mocao.org>

Childcare Support:

WOMBAT2020 has received AustMS-WIMSIG Anne Penfold Street Award to support the arrangement of childcare for the participants. If you require such support, please contact Vera Roshchina.

Organisers:

Dr Nadia Sukhorukova	Swinburne University of Technology
Dr Minh Dao	Federation University
Dr Reinier Díaz Millán	Deakin University
Prof Andrew Eberhard	RMIT University
Prof Alex Kruger	Federation University
Dr Vera Roshchina	UNSW Sydney
Dr Julien Ugon	Deakin University

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CONTENTS

Workshop Program	1
Monday 30 November 2020: Rubinov’s Day	1
Tuesday 1 December 2020	2
Wednesday 2 December 2020	3
Thursday 3 December 2020	4
Friday 4 December 2020	5
Abstracts	6
Fusheng Bai	
Surrogate Optimization for Computationally Expensive Black-box Functions	6
Hoa Bui	
Necessary Conditions for Non-Intersection of Collections of Sets	6
Regina Burachik	
Abstract Convexity and Zero Duality Gap	6
Radek Cibulka	
Ranges of nonlinear operators	7
Minh Dao	
Proximal subgradient algorithms for nonconvex nonsmooth fractional programming	7
Andrei Dmitruk	
Lagrange Multipliers Rule for a General Extremum Problem with an Infinite Number of Constraints	8
Asen Dontchev	
Teaching and Advertising Variational Analysis	8
Dmitriy Drusvyatskiy	
Active strict saddles in nonsmooth optimization	8
Joydeep Dutta	
A simple algorithm for simple bilevel programming	8
Andrew Eberhard	
Abstract Convexity and Alex Rubinov	9
Helmut Gfrerer	
On a semismooth Newton method for generalized equations	9

Barney Glover	
Alex Rubinov - A Personal Reflection	9
Sorin-Mihai Grad	
An extension of proximal point algorithms beyond convexity	10
Abderrahim Hantoute	
Nonlinear error bounds with a general class of moduli	10
Gabriel Haeser	
A new look at classical constraint qualifications for conic programming	10
Didier Henrion	
Polynomial optimization with the Lasserre hierarchy	11
Didier Henrion	
Solving numerically polynomial optimization problems	11
Nam Ho-Nguyen	
Distributionally Robust Chance-Constrained Programs under Wasserstein Ambiguity	12
Yalçın Kaya	
Path planning via maximum-information control	12
Elisabeth Köbis	
Approaches to Set Optimization in Real Linear Spaces Without Convexity Assumptions	13
Krzysztof Leśniewski	
On tangent cone to systems of inequalities and equations in Banach spaces under RCRC	13
Guoyin Li	
Convergence Rate Analysis for the Higher-Order Power Method in Best Rank One Approximation	13
Scott B Lindstrom	
Computable Centering Methods for Spiraling Algorithms and their Duals, with Motivations from the Theory of Lyapunov Functions	14
Marco A. López-Cerdá	
New representations of the normal cone to the domain of supremum functions and subdifferential calculus	14
Bruno Lourenço	
Spectral functions and an apology of Jordan Algebras	15

Russell Luke	
Convergence Analysis of the Relaxed Douglas-Rachford Algorithm	15
Daniel Mansfield	
Geometry in Ancient Mesopotamia	16
Patrick Mehlitz	
Asymptotic regularity in nonsmooth mathematical programming	16
Hossein Mohebi	
Constrained Best Approximation with Set Valued Constraints	16
Evgeni Nurminski	
Additional Cuts in Conjugate Epi-Projection Algorithm	17
Vinesha Peiris	
The extension of linear inequality method for generalised rational Chebyshev approximation	17
Chayne Planiden	
New Gradient and Hessian Approximation Methods for Derivative-free Opti- misation	18
Lindon Roberts	
Scalable Derivative-Free Optimization for Nonlinear Least-Squares Problems	18
Vera Roshchina	
Quasidifferentials: the old and the new	18
Krzysztof Rutkowski	
On autonomous ODE corresponding to Haugazeau’s algorithm	19
Claudia Sagastizábal	
Exploiting structure in nonsmooth optimization: past and present	19
Claudia Sagastizábal	
Exploiting structure in nonsmooth optimization: present and future	20
Pradeep Kumar Sharma	
Levitin-Polyak well-posedness for set optimization problems	20
Nir Sharon	
Multiscale transform for scattered manifold-valued data	20
Nadia Sukhorukova & Julien Ugon	
Quasidifferentiability and free knots spline approximation	21
Monika Syga	
Lagrangian duality for abstract convex functions	21

Matthew Tam	
Gearhart–Koshy acceleration for affine subspaces	21
Christiane Tammer	
Necessary optimality conditions in generalized convex vector optimization involving nonconvex constraints	22
Michel Théra	
Old and new results on equilibrium and quasi- equilibrium problems	22
Hong-Kun Xu	
Convergence Analysis of the Frank-Wolfe Algorithm for Convex and Nonconvex Optimization	23
Xiaoqi Yang	
Extended Newton Methods for Multiobjective Optimization: Majorizing Function Technique and Convergence Analysis	23
Jane Ye	
Second-order optimality conditions for non-convex set-constrained optimization problem	23
David Yost	
Diametrically maximal sets	24
Constantin Zalinescu	
On unconstrained optimization problems solved using CDT and triality theory	24
Alexander Zaslavski	
A turnpike property of trajectories of dynamical systems with a Lyapunov function	25

WORKSHOP PROGRAM

Monday 30 November 2020: Rubinov's Day

MORNING (08:30–12:00): Rubinov's Memorial Session Chair: Alex Kruger	
08:30–09:00	Opening session: Organisers and opening address from Prof. Barney Glover (Vice Chancellor, Western Sydney University) [video]
09:00–09:30	Barney Glover [video] Alex Rubinov - A Personal Reflection
09:30–12:00	Juan Enrique Martínez Legaz, Eldar Rubinov, Mikail Rubinov, Alex Kruger, Adil Bagirov, Regina Burachik, Yalçın Kaya, Andrew Eberhard, Jeya Jeyakumar, Boris Mordukhovich, Vladimir Gaitsgory, Nadia Sukhorukova, Julien Ugon, Xiaoqi Yang [video]
	Recorded videos: Marco A. López-Cerdá [video] , Michel Théra [video] , Gerhard-Wilhelm Weber [video] , Joydeep Dutta [video]
AFTERNOON (13:00–16:00): Rubinov's Research Session Chair: Adil Bagirov	
13:00–14:00	Andrew Eberhard [video] Abstract Convexity and Alex Rubinov
14:00–14:30	Regina Burachik [video] Abstract Convexity and Zero Duality Gap
14:30–15:30	Vera Roshchina [video] Quasidifferentials: the old and the new
15:30–16:00	Nadia Sukhorukova & Julien Ugon [video] Quasidifferentiability and free knot spline approximation
NIGHT (19:00–21:00): Rubinov's Memorial Lecture Session Master of Ceremonies: David Yost Registration	
19:00–19:30	About Professor Alex Rubinov: Professor Barney Glover, Vice Chancellor, Western Sydney University Official welcome from Federation University: Professor Chris Hutchison, Deputy Vice-Chancellor (Research and Innovation)
19:30–20:30	Daniel Mansfield The Forgotten Geometry of Mesopotamia: Revelations from an ancient clay tablet
20:30–21:00	Questions and Answers

Tuesday 1 December 2020

MORNING (09:00–12:00)	
Host: Minh Dao, Chair: Matthew Tam	
09:00–09:30	Gabriel Haeser [video] A new look at classical constraint qualifications for conic programming
09:30–10:00	Jane Ye [video] Second-order optimality conditions for non-convex set-constrained optimization problem
10:00–10:30	Fusheng Bai [video] Surrogate Optimization for Computationally Expensive Black-box Functions
10:30–11:00	Lindon Roberts [video] Scalable Derivative-Free Optimization for Nonlinear Least-Squares Problems
11:00–11:30	Hoa Bui [video] Necessary Conditions for Non-Intersection of Collections of Sets
11:30–12:00	Bruno Lourenço [video] Spectral functions and an apology of Jordan Algebras
NIGHT (19:00–21:30)	
Host: Alex Kruger, Chair: Cuong Nguyen Duy	
19:00–19:30	Marco A. López-Cerdá [video] New representations of the normal cone to the domain of supremum functions and subdifferential calculus
19:30–20:00	Elisabeth Köbis [video] Approaches to Set Optimization in Real Linear Spaces Without Convexity Assumptions
20:00–20:30	Krzysztof Rutkowski [video] On autonomous ODE corresponding to Haugazeau’s algorithm Krzysztof Rutkowski
20:30–21:00	Monika Syga [video] Lagrangian duality for abstract convex functions
21:00–21:30	Krzysztof Leśniewski [video] On tangent cone to systems of inequalities and equations in Banach spaces under RCRC

Wednesday 2 December 2020

MORNING (09:00–12:00)	
Host: Andrew Eberhard, Chair: Fusheng Bai	
09:00–10:00	Asen Dontchev (Keynote talk) [video] Teaching and Advertising Variational Analysis
10:00–10:30	Matthew Tam [video] Gearhart–Koshy acceleration for affine subspaces
10:30–11:00	Nam Ho-Nguyen [video] Distributionally Robust Chance-Constrained Programs under Wasserstein Ambiguity
11:00–11:30	Scott B Lindstrom [video] Computable Centering Methods for Spiraling Algorithms and their Duals, with Motivations from the Theory of Lyapunov Functions
11:30–12:00	Xiaoqi Yang [video] Extended Newton Methods for Multiobjective Optimization: Majorizing Function Technique and Convergence Analysis
NIGHT (19:00–22:00)	
Host: Julien Ugon, Chair: Vinesha Peiris	
19:00–20:00	Didier Henrion (Keynote lecture) [video] Polynomial optimization with the Lasserre hierarchy
20:00–20:30	Radek Cibulka [video] Ranges of nonlinear operators
20:30–21:00	Nir Sharon [video] Multiscale transform for scattered manifold-valued data
21:00–21:30	Patrick Mehlitz [video] Asymptotic regularity in nonsmooth mathematical programming
21:30–22:00	Constantin Zalinescu [video] On unconstrained optimization problems solved using CDT and triality theory

Thursday 3 December 2020

MORNING (09:00–12:00)	
Host: Vera Roshchina, Chair: Nadia Sukhorukova	
09:00–10:00	Claudia Sagastizábal (Keynote talk) [video] Exploiting structure in nonsmooth optimization: past and present
10:00–10:30	Dmitriy Drusvyatskiy [video] Active strict saddles in nonsmooth optimization
10:30–11:00	Chayne Planiden [video] New Gradient and Hessian Approximation Methods for Derivative-free Optimisation
11:00–11:30	Vinesha Peiris [video] The extension of linear inequality method for generalised rational Chebyshev approximation
11:30–12:00	David Yost [video] Diametrically maximal sets
AFTERNOON (15:00–16:00)	
Host: Andrew Eberhard, Chair: Scott Lindstrom	
15:00–15:30	Pradeep Kumar Sharma [video] Levitin-Polyak well-posedness for set optimization problems
15:30–16:00	Joydeep Dutta [video] A simple algorithm for simple bilevel programming
NIGHT (19:00–22:00)	
Host: Nadia Sukhorukova, Chair: Hoa Bui	
19:00–20:00	Didier Henrion (Keynote discussion) [video] Solving numerically polynomial optimization problems
20:00–20:30	Evgeni Nurminski [video] Additional Cuts in Conjugate Epi-Projection Algorithm
20:30–21:00	Alexander Zaslavski [video] A turnpike property of trajectories of dynamical systems with a Lyapunov function
21:00–21:30	Christiane Tammer [video] Necessary optimality conditions in generalized convex vector optimization involving nonconvex constraints
21:30–22:00	Andrei Dmitruk [video] Lagrange Multipliers Rule for a General Extremum Problem with an Infinite Number of Constraints

Friday 4 December 2020

MORNING (09:00–12:30)	
Host: Vera Roshchina, Chair: Chayne Planiden	
09:00–10:00	Claudia Sagastizábal (Keynote discussion) [video] Exploiting structure in nonsmooth optimization: present and future
10:00–10:30	Yalçın Kaya [video] Path planning via maximum-information control
10:30–11:00	Minh Dao [video] Proximal subgradient algorithms for nonconvex nonsmooth fractional programming
11:00–12:00	Hong-Kun Xu (Keynote talk) [video] Convergence Analysis of the Frank-Wolfe Algorithm for Convex and Nonconvex Optimization
12:00–12:30	Guoyin Li [video] Convergence Rate Analysis for the Higher-Order Power Method in Best Rank One Approximation
NIGHT (19:00–22:00)	
Host: Reinier Díaz Millán, Chair: Yalçın Kaya	
19:00–19:30	Abderrahim Hantoute [video] Nonlinear error bounds with a general class of moduli
19:30–20:00	Russell Luke [video] Convergence Analysis of the Relaxed Douglas-Rachford Algorithm
20:00–20:30	Hossein Mohebi [video] Constrained Best Approximation with Set Valued Constraints
20:30–21:00	Sorin-Mihai Grad [video] An extension of proximal point algorithms beyond convexity
21:00–21:30	Helmut Gfrerer [video] On a semismooth Newton method for generalized equations
21:30–22:00	Michel Théra [video] Old and new results on equilibrium and quasi- equilibrium problems

ABSTRACTS

Surrogate Optimization for Computationally Expensive Black-box Functions

FUSHENG BAI

[[video](#)]

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Optimization of computationally expensive black-box functions can be met in many areas of science, engineering, industry and business. Surrogate optimization methods use surrogate models (metamodels) to approach the real objective function with function values on limited points, and the optimal solutions of the surrogate models can be regarded as good approximations to the optimal solutions of the real objective function, thus the methods are popular in optimization of computationally expensive black-box functions. In this talk we present some new surrogate optimization methods based on the radial basis function interpolation models.

Necessary Conditions for Non-Intersection of Collections of Sets

HOA BUI

[[video](#)]

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We study elementary non-intersection properties of collections of sets, making the core of the conventional definitions of extremality and stationarity. In the setting of general Banach/Asplund spaces, we establish new primal (slope) and dual (generalized separation) necessary conditions for these non-intersection properties. The results are applied to convergence analysis of alternating projections.

Abstract Convexity and Zero Duality Gap

REGINA BURACHIK

[[video](#)]

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Using tools provided by the theory of abstract convexity, we extend conditions for zero duality gap to the context of nonconvex and nonsmooth optimization. Mimicking the classical setting, an abstract convex function is the upper envelope of a family of abstract affine functions (being conventional vertical translations of the abstract linear functions). We establish new conditions for zero duality gap under no topological assumptions on the space

of abstract linear functions. In particular, we prove that the zero duality gap property can be fully characterized in terms of an inclusion involving (abstract) ε -subdifferentials. This result is new even for the classical convex setting. Endowing the space of abstract linear functions with the topology of pointwise convergence, we extend several fundamental facts of functional/convex analysis. This includes (i) the classical Banach–Alaoglu–Bourbaki theorem (ii) the subdifferential sum rule, and (iii) a constraint qualification for zero duality gap which extends a fact established by Borwein, Burachik and Yao (2014) for the conventional convex case. As an application, we show with a specific example how our results can be exploited to show zero duality for a family of nonconvex, non-differentiable problems.

Ranges of nonlinear operators

RADEK CIBULKA

[[video](#)]

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We investigate ranges of nonlinear operators, acting in Fréchet spaces, with compact convex domains. We derive several corollaries of the abstract results. Namely, we present a Graves-type theorem in Fréchet-Montel spaces and, in finite dimensions, sufficient conditions for the directional semiregularity of a mapping defined on a (locally) convex compact set in directions from a locally conic set as well as conditions guaranteeing that the nonlinear image of a compact convex set contains a prescribed ordered interval. The presentation is based on continuing work with Marián Fabian and Tomáš Roubal.

Proximal subgradient algorithms for nonconvex nonsmooth fractional programming

MINH DAO

[[video](#)]

Federation University Australia

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In this work, we consider a broad class of nonsmooth and nonconvex fractional programs including composite optimization problems and encompassing many important modern fractional optimization problems arising from signal processing, discriminant analysis, and finance. We propose extrapolated proximal subgradient algorithms for solving this optimization model and analyze their convergence properties. The choice of our extrapolation parameter is flexible and includes the popular extrapolation parameter adopted in the restarted Fast Iterative Shrinking-Threshold Algorithm (FISTA). By providing a unified analysis framework of descent methods, we establish the convergence of the full sequence under the assumption that a suitable merit function satisfies the Kurdyka–Łojasiewicz property. Our theoretical

results are illustrated with both analytical and simulated numerical examples. This is based on joint work with R.I. Bot (Uni. Vienna) and G. Li (UNSW Sydney).

Lagrange Multipliers Rule for a General Extremum Problem with an Infinite Number of Constraints

ANDREI DMITRUK

[[video](#)]

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We consider a general optimization problem with equality and inequality constraints in a Banach space, the latter being given by closed convex cones with nonempty interiors. A necessary optimality condition in the form of Lagrange multipliers rule is presented, that is convenient for application to a wide range of optimization problems. A typical field of application is optimal control problems with state and mixed state-control constraints. This is a joint work with Nikolai Osmolovskii.

Teaching and Advertising Variational Analysis

ASEN DONTCHEV

[[video](#)]

The University of Michigan

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In this talk I would like to share my experience in teaching variational analysis in a graduate course at the mathematics department of Sofia University. I would like to also talk about my collaboration with nonmathematicians at the University of Michigan, where I have worked for a number of years on projects involving recent advances in optimization and control.

Active strict saddles in nonsmooth optimization

DMITRIY DRUSVYATSKIY

[[video](#)]

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We introduce a geometrically transparent strict saddle property for nonsmooth functions. This property guarantees that simple proximal algorithms on weakly convex problems converge only to local minimizers, when randomly initialized. We argue that the strict saddle property may be a realistic assumption in applications, since it provably holds for generic semi-algebraic optimization problems.

A simple algorithm for simple bilevel programming

JOYDEEP DUTTA

[[video](#)]

Indian Institute of Technology Kanpur

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In this talk we will discuss how a simple algorithm can be devised for minimising a convex function over the solution set of another convex optimization problem. Here we do not need to assume the Lipschitz gradient property and we have encouraging numerical results and comparisons.

Abstract Convexity and Alex Rubinov

ANDREW EBERHARD

[[video](#)]

RMIT University

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In this talk we I will give a historical account of the development of Abstract Convexity and relate areas that are precursors found in Generalised Convexity. This talk will contain some personal recollections of work and conversation with Alex Rubinov. We discuss Alex's perspectives on global vs local optimisation some results that have followed after Alex's passing.

On a semismooth Newton method for generalized equations

HELMUT GFRENER

[[video](#)]

Johannes Kepler University Linz, Austria

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In this talk we introduce a new notion of semismoothness which pertains both sets as well as multifunctions. In the case of single-valued maps it is closely related with the standard notion of semismoothness introduced by Qi and Sun in 1993. Semismoothness can be equivalently characterized in terms of regular, limiting and directional limiting coderivatives. Then we present a semismooth Newton method for solving inclusions and generalized equations (GEs), where the linearization concerns both the single-valued and the multi-valued part and it is performed on the basis of the respective coderivatives. We also present encouraging results of some numerical experiments.

Alex Rubinov - A Personal Reflection

BARNEY GLOVER

[video]

Western Sydney University

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This presentation will take the form of a personal reflection on the presenter's relationship with Alex Rubinov from the late 1980's covering Alex's first visit to Australia and his eventual appointment to the University of Ballarat. The joint research in abstract functional analysis, duality, solvability theory, abstract convexity and global optimisation will be mentioned as well areas of applications. Hopefully the talk will highlight the immense contribution of Alex Rubinov, his intellect and his humanity.

An extension of proximal point algorithms beyond convexity

SORIN-MIHAI GRAD

[video]

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Motivated by some recent existence results for mixed variational inequalities where no convexity hypotheses were imposed on the involved function, we introduce and investigate a new generalized convexity notion for functions called prox-convexity, whose definition is satisfied, for instance, by some classes of quasiconvex functions and weakly convex functions. We show that the classical proximal point algorithm remains convergent when the convexity of the function to be minimized is relaxed to prox-convexity and the same happens with the recent Golden-Ratio algorithm for solving mixed variational inequalities due to Malitsky. We also present some numerical experiments that confirm the theoretical results. The talk is based on joint work with Felipe Lara (University of Tarapaca, Arica, Chile).

Nonlinear error bounds with a general class of moduli

ABDERRAHIM HANTOUTE

[video]

Universidad of Alicante

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Existence of nonlinear error bounds is proved, while the associated moduli do not need to be increasing or continuous. The techniques are based on a variational approach, applied in suitable complete metric spaces that are close to the original metric. The explicit form of such metrics and the calculation of the associated strong slopes allow the quantification of such error bounds.

A new look at classical constraint qualifications for conic programming

GABRIEL HAESER

[[video](#)]

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Nondegeneracy and Robinson's condition are the most relevant and useful constraint qualifications for conic programming. In the case of nonlinear semidefinite programming, they are defined by fixing an eigenvector basis of the, let us say, r -dimensional kernel of the constraint matrix and requiring the (positive) linear independence of a set of $r(r+1)/2$ derivative vectors. We show that by considering all eigenvector bases of the kernel, the (positive) linear independence requirement can be equivalently made in a smaller set of r derivative vectors. This allows us to identify that not all bases are relevant for a constraint qualification to be defined, giving rise to strictly weaker variants of nondegeneracy and Robinson's condition. In particular, our weaker variant of nondegeneracy reduces to the linear independence constraint qualification for nonlinear programming when considering a diagonal semidefinite matrix. More generally, when the problem has a diagonal block structure, the conditions formulated as a single matrix constraint is equivalent to the one formulated with several semidefinite matrices.

Polynomial optimization with the Lasserre hierarchy

DIDIER HENRION

[[video](#)]

LAAS-CNRS Univ. Toulouse, France and Czech Tech. Univ. Prague, Czechia

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This is an elementary introduction to the Lasserre hierarchy applied to polynomial optimization. We show how the non-convex non-linear problem of minimizing a multivariate polynomial subject to polynomial inequality constraints can be reformulated as a family of convex semidefinite optimization problems of increasing size. Instrumental to this reformulation is the duality between the cone of positive polynomials and the cone of moments of positive measures, and the approximations of these cones with sums of squares certificates of positivity.

Solving numerically polynomial optimization problems

DIDIER HENRION

[[video](#)]

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With the help of numerical examples using the Matlab parser GloptiPoly interfaced with the conic solver MOSEK, we illustrate how polynomial optimization problems can be solved efficiently with the Lasserre hierarchy. We describe certificates of global optimality based on flat extensions of moment matrices, as well as approximate recovery of the variety of global minimizers with the Christoffel-Darboux polynomial.

Distributionally Robust Chance-Constrained Programs under Wasserstein Ambiguity

NAM HO-NGUYEN

[[video](#)]

Discipline of Business Analytics, The University of Sydney

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We consider exact deterministic mixed-integer programming (MIP) reformulations of distributionally robust chance-constrained programs (DR-CCP) over Wasserstein ambiguity sets. The existing MIP formulations are known to have weak continuous relaxation bounds, and, consequently, for hard instances with small radius, or with large problem sizes, the branch-and-bound based solution processes suffer from large optimality gaps even after hours of computation time. This significantly hinders the practical application of the DR-CCP paradigm. Motivated by these challenges, we conduct a polyhedral study to strengthen these formulations. We reveal several hidden connections between DR-CCP and its nominal counterpart (the sample average approximation), mixing sets, and robust 0-1 programming. By exploiting these connections in combination, we provide an improved formulation and two classes of valid inequalities for DR-CCP. We test the impact of our results on a stochastic transportation problem numerically. Our experiments demonstrate the effectiveness of our approach; in particular our improved formulation and proposed valid inequalities reduce the overall solution times remarkably. Moreover, this allows us to significantly scale up the problem sizes that can be handled in such DR-CCP formulations by reducing the solution times from hours to seconds.

Path planning via maximum-information control

YALÇIN KAYA

[video]

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We are concerned with finding an optimal path for an observer, or sensor, which is to estimate the position of a target, using bearing angle measurements. The generated path is optimal in the sense that, along the path, information, and thus the efficiency of an estimator employed, is maximized. In other words, an observer path is deemed to be optimal if it maximizes information so that the location of the target is estimated with smallest uncertainty in some sense. We formulate this as an optimal control problem maximizing the determinant of the Fisher information matrix, which is one of the measures of information. We carry out numerical experiments and discuss the solutions obtained, by means of the maximum principle.

Approaches to Set Optimization in Real Linear Spaces Without Convexity Assumptions

ELISABETH KÖBIS

[video]

Norwegian University of Science and Technology (NTNU), Norway)

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Set optimization is concerned with finding minimal solutions of set-valued mappings, where the outcome sets are compared by binary relations among sets, so-called set relations. In this talk we give a characterization of several set relations by means of nonlinear scalarization functionals in real linear spaces. Our methods do not rely on any convexity assumptions on the considered sets.

On tangent cone to systems of inequalities and equations in Banach spaces under RCRC

KRZYSZTOF LEŚNIEWSKI

[video]

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Under the relaxed constant rank condition, introduced by Minchenko and Stakhovskii, we prove that linearized cone is contained in the tangent cone for sets represented as solution sets to systems of finite numbered inequalities and equations given by C^1 functions defined on Banach spaces. This is joint work with E. Bednarczuk and K. Rutkowski.

Convergence Rate Analysis for the Higher-Order Power Method in Best Rank One Approximation

GUOYIN LI

[video]

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A popular and classical method for finding the best rank-one approximation of a real tensor is the higher order power method (HOPM). In this talk, we first provide an explicit sublinear convergence rate for HOPM, in terms of the dimension/order of the tensor space. Then, we show that HOPM converges R-linearly for orthogonally decomposable tensors with order at least 3, and “typically” exhibits a global R-linear convergence rate for general tensors in a suitable sense.

Computable Centering Methods for Spiraling Algorithms and their Duals, with Motivations from the Theory of Lyapunov Functions

SCOTT B LINDSTROM

[video]

The Hong Kong Polytechnic University

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Circumcentering reflection methods (CRMs) have been shown to obviate spiraling for Douglas–Rachford Method (DR) for certain feasibility problems. I will show how CRMs, subgradient projections, and Newton–Raphson are all describable as gradient-based methods for minimizing Lyapunov functions constructed for DR operators, with the former returning the minimizers of quadratic surrogates for the Lyapunov function. I will then introduce a new centering method that shares these properties with the added advantages that it: 1) does not rely on subproblems (e.g. reflections) and so may be applied for any operator whose iterates spiral; 2) provably has the aforementioned Lyapunov properties with few structural assumptions and so is generically suitable for primal/dual implementation; and 3) maps spaces of reduced dimension into themselves whenever the original operator does. This makes possible the first primal/dual implementation of a centering method for accelerating Alternating Direction Method of Multipliers. I’ll show how it works.

New representations of the normal cone to the domain of supremum functions and subdifferential calculus

MARCO A. LÓPEZ-CERDÁ

[video]

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In the first part of the talk we present new characterizations of the normal cone to the effective domain of the supremum of an arbitrary family of convex functions. These results are applied in the second part to give new formulas for the subdifferential of the supremum function, which use both the active and nonactive functions at the reference point, the last ones multiplied by some appropriate parameters. In contrast with previous works in the literature, the main feature of our approach is that the normal cone to the effective domain of the supremum (or to finite-dimensional sections of this domain) does not appear in the formulas. A new type of optimality conditions for convex optimization is also established as a consequence. The presentation concerns the results established in a recent paper coauthored by A. Hantoute and R. Correa.

Spectral functions and an apology of Jordan Algebras

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[video]

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Jordan Algebras are a fixture of the optimization field, although not necessarily an universally loved one. In fact, I believe Jordan Algebras might be the vegemite of our community. And, much like the aforementioned spread, I'll try to make the case that Jordan Algebras have their (research) benefits, even if they can be an acquired taste. I will illustrate this with recent results on generalized subdifferentials of spectral functions, which are real functions that only depend on the eigenvalues of their input. This is a joint work with Akiko Takeda (U. Tokyo). Preprint link: <https://arxiv.org/abs/1902.05270>.

Convergence Analysis of the Relaxed Douglas-Rachford Algorithm

RUSSELL LUKE

[video]

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Motivated by nonconvex, inconsistent feasibility problems in imaging, the relaxed alternating averaged reflections algorithm, or relaxed Douglas-Rachford algorithm (DR λ), was

first proposed over a decade ago. Convergence results for this algorithm are limited either to convex feasibility or consistent nonconvex feasibility with strong assumptions on the regularity of the underlying sets. Using an analytical framework depending only on metric subregularity and pointwise almost averagedness, we analyze the convergence behavior of DR λ for feasibility problems that are both nonconvex and inconsistent. We introduce a new type of regularity of sets, called super-regular at a distance, to establish sufficient conditions for local linear convergence of the corresponding sequence. These results subsume and extend existing results for this algorithm.

Geometry in Ancient Mesopotamia

DANIEL MANSFIELD

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Mathematics is said to have begun long ago in ancient Mesopotamia, also known as the cradle of civilization. This culture had slipped into legend until the late 19th century when archaeologists uncovered thousands upon thousands of clay documents from lost cities such as Sippar, Lasa, and Babylon. Amongst these were mathematical and geometric texts, many are yet to be translated and some are surprisingly advanced. This talk is an introduction to their unique form of arithmetic and geometry, including some very recent discoveries.

Asymptotic regularity in nonsmooth mathematical programming

PATRICK MEHLITZ

[video]

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Based on the tools of limiting variational analysis, we derive a sequential necessary optimality condition for nonsmooth mathematical programs which holds without any additional assumptions. In order to ensure that stationary points in this new sense are already Mordukhovich-stationary, the presence of a constraint qualification which we call AM-regularity is necessary. We investigate the relationship between AM-regularity and other constraint qualifications from nonsmooth optimization like metric (sub-)regularity of the underlying feasibility mapping. Our findings are applied to optimization problems with geometric and, particularly, disjunctive constraints. This way, it is shown that AM-regularity recovers recently introduced cone-continuity-type constraint qualifications, sometimes referred to as AKKT-regularity, from standard nonlinear and complementarity-constrained optimization. Finally, we discuss some consequences of AM-regularity for the limiting variational calculus.

Constrained Best Approximation with Set Valued Constraints

HOSSEIN MOHEBI

[video]

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In this talk, we employ generalized subdifferential closedness condition and generalized Guignard's constraint qualification to present a dual cone characterization of a constraint set S with C -convex set valued mappings, where C is a non-empty closed convex cone in \mathbb{R}^n . The obtained results provide sufficient conditions for which the "strong conical hull intersection property" (strong CHIP) holds. Moreover, we establish necessary and sufficient conditions for characterizing "perturbation property" of the best approximation $p \in S$ to any point $x \in \mathbb{R}^n$ from a convex set $S := K \cap S$ by the strong CHIP of K and S at the point p , where K is a non-empty closed convex set in \mathbb{R}^n . In the special case, where the set S is convex, we derive the Lagrange multiplier characterizations of constrained best approximation under generalized subdifferential closedness condition and generalized Guignard's constraint qualification. The clarification of our results is illustrated by numerical experiments.

Additional Cuts in Conjugate Epi-Projection Algorithm

EVGENI NURMINSKI

[video]

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This talk will discuss the computational effects of the additional down-cuts in convex optimization's conjugate epi-projection algorithm. These cuts introduce additional low-dimensional auxiliary optimization in the algorithm and improve its relaxational properties.

The extension of linear inequality method for generalised rational Chebyshev approximation

VINESHA PEIRIS

[video]

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In this talk we will demonstrate the correspondence between the linear inequality method developed for rational Chebyshev approximation and the bisection method used in quasiconvex optimisation. It naturally connects rational and generalised rational Chebyshev approximation problems with modern developments in the area of quasiconvex functions. Moreover, the linear inequality method can be extended to a broader class of Chebyshev approximation

problems, where the corresponding objective functions remain quasiconvex.

New Gradient and Hessian Approximation Methods for Derivative-free Optimisation

CHAYNE PLANIDEN

[\[video\]](#)

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In general, derivative-free optimisation (DFO) uses approximations of first- and second-order information in minimisation algorithms. DFO is found in direct-search, model-based, trust-region and other mainstream optimisation techniques and is gaining popularity in recent years. This work discusses previous results on some particular uses of DFO: the proximal bundle method and the VU-algorithm, and then presents improvements made this year on the gradient and Hessian approximation techniques. These improvements can be inserted into any routine that requires such estimations.

Scalable Derivative-Free Optimization for Nonlinear Least-Squares Problems

LINDON ROBERTS

[\[video\]](#)

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Derivative-free optimisation (DFO) methods are an important class of optimisation routines with applications in areas such as in image analysis and data science. However, in model-based DFO methods, the computational cost of constructing local models can be high, particularly for large-scale problems. Considering nonlinear least-squares problems, we improve on state-of-the-art DFO by performing dimensionality reduction in the observational space using sketching methods, avoiding the construction of a full local model. Our approach has a per-iteration computational cost which is linear in problem dimension in a big data regime, and numerical evidence demonstrates that, compared to existing software, it has dramatically improved runtime performance on overdetermined least-squares problems. This is joint work with Coralia Cartis and Tyler Ferguson (University of Oxford).

Quasidifferentials: the old and the new

VERA ROSHCINA

[video]

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I will give an overview of the calculus of quasidifferentials, covering both historical developments and more recent results.

The talk will include a pre-recorded interview with Prof. Lyudmila Polyakova.

On autonomous ODE corresponding to Haugazeau's algorithm

KRZYSZTOF RUTKOWSKI

[video]

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In this presentation we define an ODE corresponding to Haugazeau's Algorithm in Hilbert space settings. We show that under mild conditions for any non-stationary initial point, the vector field given in corresponding ODE is locally Lipschitz. Then we show existence and uniqueness of solutions to ODE for some period of time. Using the property that the trajectories may only blow up or can be extended we will show that the existence and uniqueness of solutions can be shown on half-line.

Exploiting structure in nonsmooth optimization: past and present

CLAUDIA SAGASTIZÁBAL

[video]

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In many optimization problems nonsmoothness appears in a structured manner, because the objective function has composite or separable form. The first type of structure is found in regularized least-squares problems arising in compressed sensing and machine-learning areas. The second type often results from applying some decomposition technique to problems that cannot be solved directly. This context is frequent in stochastic programming, bilevel optimization, equilibrium problems.

The talk will give a panorama of techniques that have proven successful in exploiting structural properties that are somewhat hidden behind nonsmoothness. Such is the case of two approaches that we shall revisit, without getting into technical details: a log-barrier Tikhonov smoothing and the so-called VU-space decomposition. Throughout the presentation the emphasis is put on transmitting the main ideas and concepts, illustrating with

examples the presented material. Credit to co-authors will be given along the talk.

Exploiting structure in nonsmooth optimization: present and future

CLAUDIA SAGASTIZÁBAL

[[video](#)]

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Thanks to the VU-decomposition, it is possible to make a Newton-like move in certain subspace, that concentrates all the function “smoothness”, at least locally. On its orthogonal complement, the “sharp” V -subspace, an intermediate iterate is defined in such a way that the overall convergence is driven by the U -step.

The proximal variants of bundle methods provide the ingredients needed to build superlinearly convergent algorithms in nonsmooth optimization. For functions whose proximal points are easy to compute, as in Machine Learning, an implementable quasi-Newton VU-algorithm that is superlinearly convergent is achievable. In order to trigger a fruitful discussion and exchanges with the participants, in this second part of the presentation we shall outline a possible approach along those lines.

Levitin-Polyak well-posedness for set optimization problems

PRADEEP KUMAR SHARMA

[[video](#)]

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In this presentation, we discuss different kinds of Levitin-Polyak well-posedness for set optimization problems and their relationships with respect to the set order relations defined by Minkowski’s difference in the family of bounded sets. Furthermore, by using the Kuratowski measure of noncompactness, we give some characterizations of Levitin-Polyak well-posedness for set optimization problems. Moreover, we establish the relationship between stability and LP well-posedness of set optimization problem by defining approximating solution maps. Several examples will be given in support of concepts and results of this presentation.

Multiscale transform for scattered manifold-valued data

NIR SHARON

[[video](#)]

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A multiscale transform is a standard tool in signal and image processing that enables a

hierarchical analysis of an object mathematically. Customarily, the first scale in the transform corresponds to a coarse representation, and as scales increase, so is the refinement level of the approximation. The multiscale approach introduces a dynamic and flexible framework that presents many computational and approximation advantages. In the talk, we will introduce a multiscale method to approximate manifold-valued functions. These functions appear in various applications, for example, Diffusion-MRI imaging, simulation using Reduced Order Model (ROM), and more. We will introduce our multiscale construction and show some of its theoretical properties. Then, we will conclude with some numerical examples and future directions, as time permits.

Quasidifferentiability and free knots spline approximation

NADIA SUKHORUKOVA & JULIEN UGON [\[video\]](#)

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The problem of approximating a continuous function by a piecewise polynomial (polynomial spline) has been studied for over several decades; yet, when the knots joining the polynomials are also variable, finding conditions for a best Chebyshev approximation remains an open problem. We derive a necessary optimality condition for a best approximation which is stronger than the existing ones. We were able to improve the existing results using Demyanov-Rubinov quasidifferentials.

Lagrangian duality for abstract convex functions

MONIKA SYGA [\[video\]](#)

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We use the Φ -convexity theory and minimax theorems for Φ -convex functions to investigate Lagrangian duality for nonconvex optimization problems. In particular we provide the necessary and sufficient conditions for the zero duality gap.

Gearhart–Koshy acceleration for affine subspaces

MATTHEW TAM [\[video\]](#)

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The method of cyclic projection is an iterative scheme which can be used to find nearest points in the intersection of finitely many affine subspaces, having access only to the orthog-

onal projectors onto the individual subspaces. In an attempt to accelerate its convergence, Gearhart and Koshy (1989) proposed a modification of the scheme which, in each iteration, performs an exact line search based on minimising the distance to the desired nearest point. In the special case when the affine subspaces are linear subspaces, this line search can be made explicit by using the fact that the zero vector is always feasible. In this talk, I will discuss an alternate (but nevertheless still explicit) implementation of this line search which does not require knowledge of a feasible vector, thus providing an efficient implementation of the scheme for affine subspaces.

Necessary optimality conditions in generalized convex vector optimization involving nonconvex constraints

CHRISTIANE TAMMER

[video]

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The aim of this talk is to derive necessary optimality conditions for efficient solutions of vector optimization problems involving not necessarily convex constraints. In a paper by Guenther and Tammer (2018), it is shown that the set of efficient solutions of a constrained vector optimization problem can be computed completely by considering two corresponding unconstrained vector optimization problems. By employing these results and by applying methods of generalized differentiation, we show that it is possible to derive necessary optimality conditions for a problem with a nonconvex feasible set. These optimality conditions have a simple structure because the normal cone with respect to the constraints is not involved. Finally, we apply our results to vector-valued approximation problems with a not necessarily convex feasible set and derive necessary optimality conditions.

Old and new results on equilibrium and quasi- equilibrium problems

MICHEL THÉRA

[video]

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In this talk I will briefly survey some old results which are going back to Ky Fan and Brezis-Nirenberg and Stampacchia. Then I will give some new results related to the existence of solutions to equilibrium and quasi- equilibrium problems without any convexity assumption. Coverage includes some equivalences to the Ekeland variational principle for bifunctions and basic facts about transfer lower continuity. An application is given to systems of quasi-equilibrium problems.

Convergence Analysis of the Frank-Wolfe Algorithm for Convex and Nonconvex Optimization

HONG-KUN XU

[video]

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The Frank-Wolfe algorithm (FWA), a very first optimization algorithm and also known as the conditional gradient algorithm, was introduced by Marguerite Frank and Philip Wolfe in 1956. Due to its simple linear subproblems, FWA has recently been paid much attention to solve constrained optimization problems over closed convex bounded sets. The convergence of FWA depends on the notion of the curvature constant of the objective function. In this talk we will extend this notion to that of the curvature constant of order $\sigma \in (1, 2]$. Basing upon this notion we obtain the convergence rate of FWA in the case where the objective function is not necessarily Lipschitz continuous and the stepsizes are chosen either by the descent line search or by the open loop rule. Both convex and nonconvex cases will be investigated.

Extended Newton Methods for Multiobjective Optimization: Majorizing Function Technique and Convergence Analysis

XIAOQI YANG

[video]

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We consider the extended Newton method for approaching a Pareto optimum of a multiobjective optimization problem, establish quadratic convergence criteria, and estimate a radius of convergence ball under the assumption that the Hessians of objective functions satisfy an L -average Lipschitz condition. These convergence theorems significantly improve the corresponding ones in [J. Fliege, L. M. G. Drummond, and B. F. Svaiter, SIAM J. Optim., 20 (2009), pp. 602–626]. As applications of the obtained results, convergence theorems under the classical Lipschitz condition or the γ -condition are presented for multiobjective optimization, and the global quadratic convergence results of the extended Newton method with Armijo/Goldstein/Wolfe line-search schemes are also provided.

Second-order optimality conditions for non-convex set-constrained optimization problem

JANE YE

[video]

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In this paper we study second-order optimality conditions for non-convex set-constrained optimization problems. For a convex set-constrained optimization problem, it is well-known that second-order optimality conditions involve the support function of the second-order tangent set. In this paper we propose two approaches for establishing second-order optimality conditions for the non-convex case. In the first approach we extend the concept of the support function so that it is applicable to general non-convex set-constrained problems, whereas in the second approach we introduce the notion of the directional regular tangent cone and apply classical results of convex duality theory. Besides the second-order optimality conditions, the novelty of our approach lies in the systematic introduction and use, respectively, of directional versions of well-known concepts from variational analysis.

Diametrically maximal sets

DAVID YOST

[video]

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A set is diametrically maximal if the addition of any extra point strictly increases its diameter. This talk will survey their basic properties, in Euclidean spaces and non-Euclidean spaces, indicate their relationship to optimisation and other branches of mathematics, and present some recent joint work with Elisabetta Maluta.

On unconstrained optimization problems solved using CDT and triality theory

CONSTANTIN ZALINESCU

[video]

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DY Gao solely or in collaboration applied his Canonical duality theory (CDT) for solving a class of unconstrained optimization problems, getting the so-called “trinality theorems”.

As mentioned in the recent book [GLR17], “trinality theory can be used to identify both global and local optimality conditions and to develop powerful algorithms for solving chal-

lenging problems in complex systems”.

Unfortunately, the “double-min duality” from the triality theorems published before 2010 revealed to be false, even if in the paper [G03] DY Gao announced that “certain additional conditions” are needed for getting it.

Beginning with 2011, DY Gao together with some of his collaborators published several papers in which they added additional conditions for getting “double-min” and “double-max” dualities in the triality theorems; a general case is considered in [GW17], while particular cases are treated in several papers.

Our aim in this talk is to indicate an approach for treating rigorously this kind of problems and to discuss several results concerning the “triality theory” obtained up to now.

References

[G03] DY Gao, Perfect duality theory and complete solutions to a class of global optimization problems, *Optimization*, 52 (2003), 467-493.

[GLR17] DY Gao, V Latorre, N Ruan, Preface. In: *Canonical Duality Theory. Unified Methodology for Multidisciplinary Study*, DY Gao et al. (eds), *Advances in Mechanics and Mathematics* 37. Cham (Switzerland), Springer (2017).

[GW17] DY Gao, C Wu, Triality theory for general unconstrained global optimization problems, *ibidem*, pp. 127-153.

A turnpike property of trajectories of dynamical systems with a Lyapunov function

ALEXANDER ZASLAVSKI

[video]

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In this talk we discuss the structure of trajectories of discrete disperse dynamical systems with a Lyapunov function which are generated by set-valued mappings. This class of dynamical systems was introduced by A. M. Rubinov in 1980. We establish a version of the turnpike property which hold for all trajectories of such dynamical systems which are of a sufficient length. This result is usually true for models of economic growth which are prototypes of our dynamical systems.